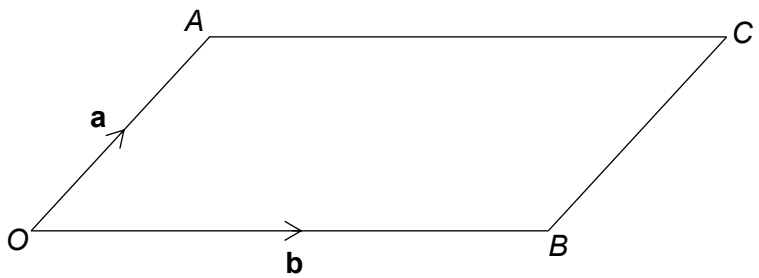
Vectors

SCAN ME

REVISE THIS TOPIC

1 *OACB* is a parallelogram.



$\overrightarrow{OA}$ = **a** $\overrightarrow{OB}$ = **b**

Write the following vectors in terms of **a** and **b**.

1 (a) $\overrightarrow{AO}$ **[1 mark]**

Answer $-a$

1 (b) $\overrightarrow{BC}$ **[1 mark]**

Answer a

1 (c) $\overrightarrow{AB}$ **[1 mark]**

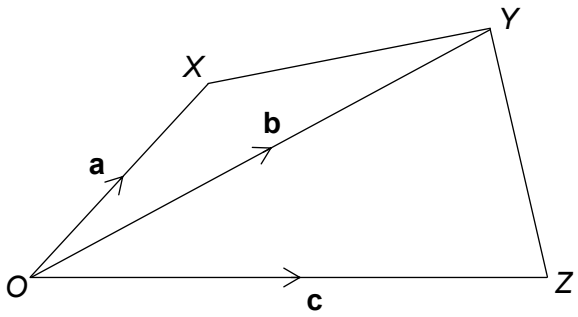
Answer $b - a$

1 (d) $\overrightarrow{CO}$ **[1 mark]**

Answer $-a - b$

4

2 *OXYZ* is a quadrilateral.



$\overrightarrow{OX}$ = **a** $\overrightarrow{OY}$ = **b** $\overrightarrow{OZ}$ = **c**

Write the following vectors in terms of **a, b** and **c**.

2 (a) $\overrightarrow{ZO}$ **[1 mark]**

Answer -c

2 (b) $\overrightarrow{XY}$ **[1 mark]**

Answer b - a

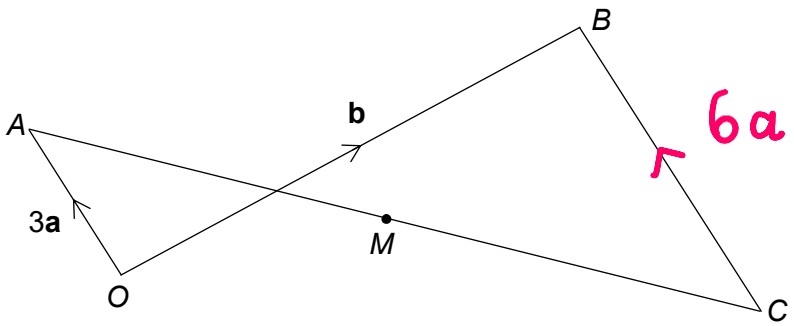
2 (c) $\overrightarrow{ZY}$ **[1 mark]**

Answer b - c

2 (d) $\overrightarrow{XZ}$ **[1 mark]**

Answer c - a

3



$\overrightarrow{OA} = 3\mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$ $\overrightarrow{CB} = 2\overrightarrow{OA}$

Write the following vectors in terms of **a, b** and **c**.

3 (a) $\overrightarrow{AB}$ **[1 mark]**

Answer $b - 3a$

3 (b) $\overrightarrow{CA}$ **[2 marks]**

$\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BO} + \overrightarrow{OA}$

$= 6a - b + 3a$

Answer $9a - b$

3 (c) M is the midpoint of AC.

Write $\overrightarrow{CM}$ in terms of **a** and **b**. **[2 marks]**

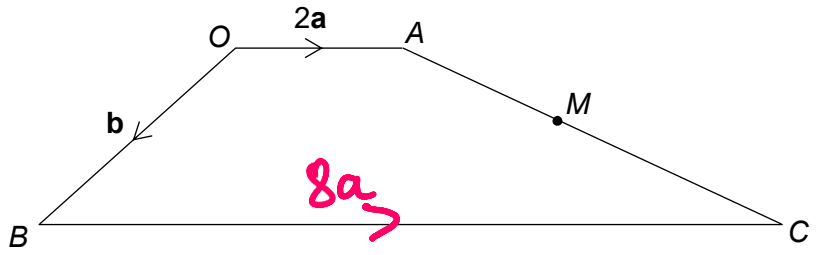
$\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CA}$

$= \frac{1}{2}(9a - b)$

Answer $\frac{9}{2}a - \frac{1}{2}b$

9

4 *OACB* is a trapezium



$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$ $\overrightarrow{BC} = 4\overrightarrow{OA}$

4 (a) Write $\overrightarrow{AC}$ in term of **a** and **b**. **[2 marks]**

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BC}$

$= -2a + b + 8a$

Answer $6a + b$

4 (b) *M* is the midpoint of *AC*.

Write $\overrightarrow{BM}$ in term of **a** and **b**. **[3 marks]**

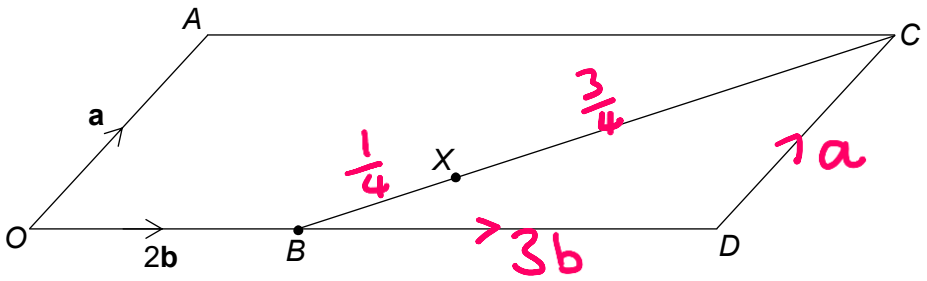
$\overrightarrow{BM} = \overrightarrow{BO} + \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$

$= -b + 2a + \frac{1}{2}(6a + b)$

$= -b + 2a + 3a + \frac{1}{2}b$

Answer $5a - \frac{1}{2}b$

5 *OACD* is a parallelogram.



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = 2\mathbf{b}$ $\overrightarrow{OD} = 2.5\overrightarrow{OB}$

5 (a) Write $\overrightarrow{AD}$ in term of **a** and **b**. **[2 marks]**

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$= -a + 5b$

Answer $5b - a$

5 (b) Write $\overrightarrow{BC}$ in term of **a** and **b**. **[2 marks]**

$\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC}$

$= 3b + a$

Answer $3b + a$

5 (c) $BX : XC = 1 : 3$

Write $\overrightarrow{OX}$ in term of **a** and **b**. **[2 marks]**

$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX}$

$= 2b + \frac{1}{4}\overrightarrow{BC}$

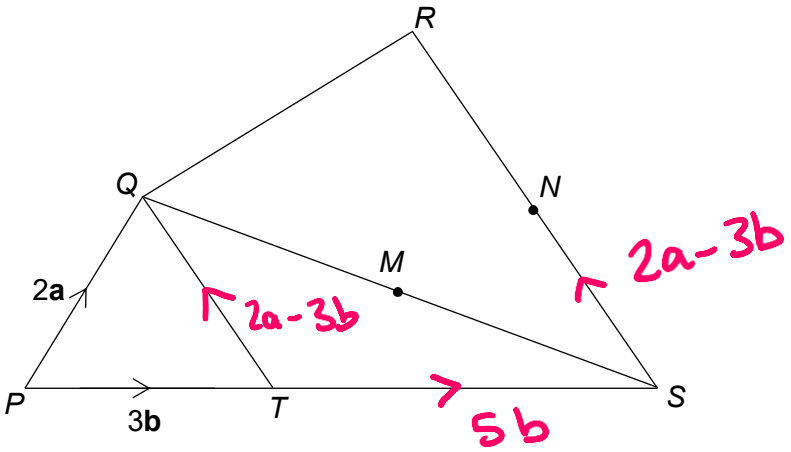
$= 2b + \frac{1}{4}(3b + a)$

$= 2b + \frac{3}{4}b + \frac{1}{4}a$

Answer $\frac{11}{4}b + \frac{1}{4}a$

11

6 *PQRST* is a pentagon.



$\overrightarrow{PQ} = 2\mathbf{a}$ $\overrightarrow{PT} = 3\mathbf{b}$ $\overrightarrow{RS} = 2\overrightarrow{QT}$

PTS is a straight line with $PT : TS = 3 : 5$

$\uparrow TS = 5b$

M is the midpoint of *QS*.
N is the midpoint of *RS*.

Write $\overrightarrow{MN}$ in term of **a** and **b**. **[4 marks]**

$\overrightarrow{TQ} = 2a - 3b$ $\overrightarrow{RS} = 2(2a - 3b)$

$\overrightarrow{SN} = 2a - 3b$ $= 4a - 6b$

$\overrightarrow{QS} = \overrightarrow{QT} + \overrightarrow{TS}$

$= 3b - 2a + 5b$

$= 8b - 2a$

$\overrightarrow{MS} = \frac{1}{2}(8b - 2a)$

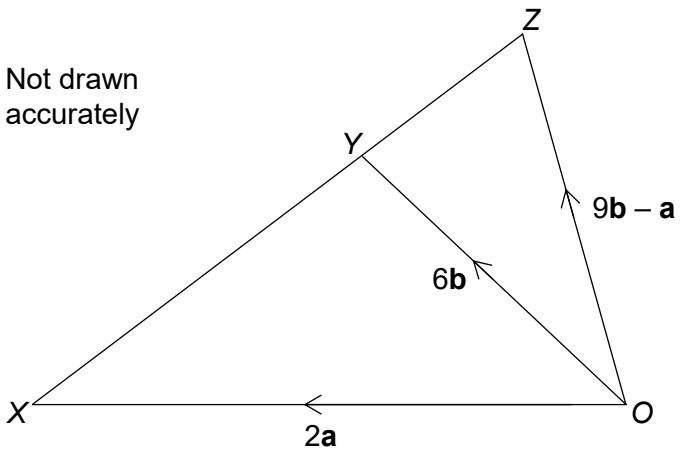
$= 4b - a$

$\overrightarrow{MN} = \overrightarrow{MS} + \overrightarrow{SN}$

$= 4b - a + 2a - 3b$

Answer $a + b$

7



Prove, using vectors, that *XYZ* is a straight line. **[3 marks]**

$\overrightarrow{XY} = 6b - 2a$

$\overrightarrow{XZ} = 9b - a - 2a$

$= 9b - 3a$

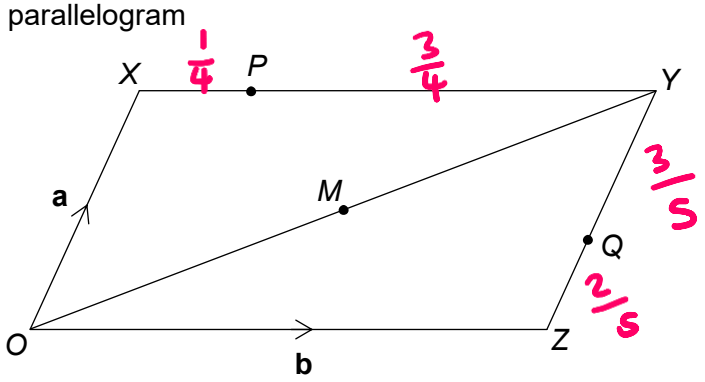
$= \frac{3}{2}(6b - 2a)$

$\overrightarrow{XZ} = \frac{3}{2}XY$

therefore XYZ is a straight line

7

8 $OXYZ$ is a parallelogram



$\overrightarrow{OX} = \mathbf{a}$ $\overrightarrow{OZ} = \mathbf{b}$

$XP : PY = 1 : 3$
$ZQ : QY = 2 : 3$

M is the midpoint of OY

8 (a) Write $\overrightarrow{PQ}$ in term of **a** and **b**. **[2 marks]**

$\overrightarrow{PQ} = \overrightarrow{PY} + \overrightarrow{YQ}$
$= \frac{3}{4}b - \frac{3}{5}a$

Answer $\frac{3}{4}b - \frac{3}{5}a$

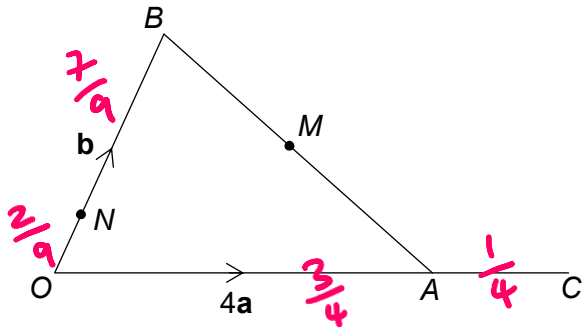
8 (b) Write $\overrightarrow{MQ}$ in term of **a** and **b**. **[3 marks]**

$\overrightarrow{MQ} = \overrightarrow{MY} + \overrightarrow{YQ}$
$= \frac{1}{2}(OY) + \overrightarrow{YQ}$
$= \frac{1}{2}(a+b) - \frac{3}{5}a$
$= \frac{1}{2}a + \frac{1}{2}b - \frac{3}{5}a$

$\frac{1}{2}b - \frac{1}{10}a$

Answer $\frac{1}{2}b - \frac{1}{10}a$

9



$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$OA : OC = 3 : 4$
$ON : OB = 2 : 9$

M is the midpoint of AB

9 (a) Write $\overrightarrow{MC}$ in term of **a** and **b**. **[3 marks]**

$\overrightarrow{MC} = \overrightarrow{MA} + \overrightarrow{AC}$
$= \frac{1}{2}\overrightarrow{BA} + \overrightarrow{AC}$
$= \frac{1}{2}(4a - b) + \frac{4}{3}a$
$= 2a - \frac{1}{2}b + \frac{4}{3}a$

Answer $\frac{10}{3}a - \frac{1}{2}b$

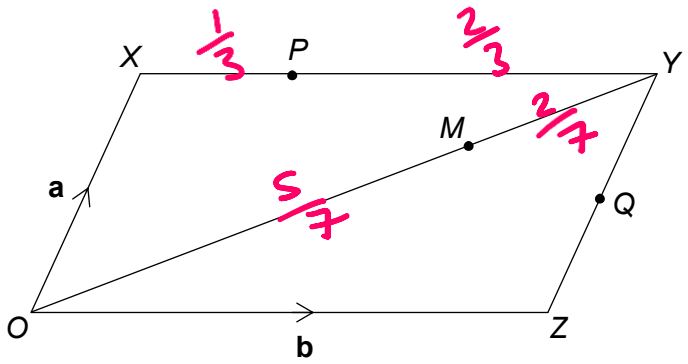
9 (b) Write $\overrightarrow{NM}$ in term of **a** and **b**. **[2 marks]**

$\overrightarrow{NM} = \overrightarrow{NB} + \overrightarrow{BM}$
$= \frac{7}{9}b + 2a - \frac{1}{2}b$

Answer $\frac{5}{18}b + 2a$

10

10 *OXYZ* is a parallelogram



Not drawn accurately

$\overrightarrow{OX} = \mathbf{a}$ $\overrightarrow{OZ} = \mathbf{b}$

$ZQ = QY$
$XP : PY = 1 : 2$
$OM : MY = 5 : 2$

Prove, using vectors, that PMQ is a straight line. **[4 marks]**

$\overrightarrow{PM} = \overrightarrow{PY} + \overrightarrow{YM}$

$= \overrightarrow{PY} + \frac{2}{7}(\overrightarrow{YO})$

$= \frac{2}{3}b + \frac{2}{7}(-b-a)$

$= \frac{2}{3}b - \frac{2}{7}b - \frac{2}{7}a$

$= \frac{8}{21}b - \frac{2}{7}a$

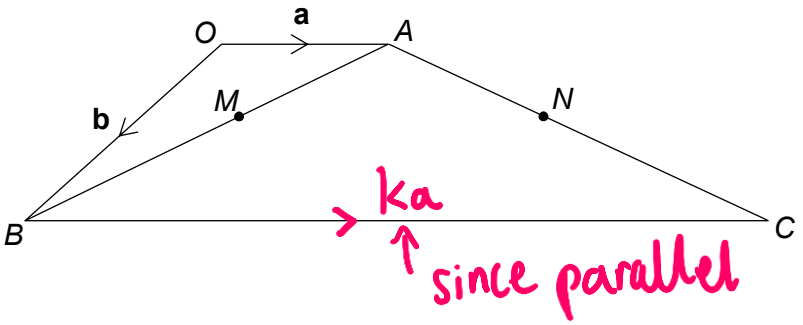
$\overrightarrow{PQ} = \overrightarrow{PY} + \overrightarrow{YQ}$

$= \frac{2}{3}b - \frac{1}{2}a$

$\overrightarrow{PQ} = \frac{7}{4}\overrightarrow{PM}$

therefore PMQ is a straight line

11 *OACB* is a trapezium



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

M and *N* are the midpoints of *AB* and *AC*.

Prove, using vectors, that *MN* is parallel to *OA*. **[4 marks]**

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$

$= \frac{1}{2}\overrightarrow{BA} + \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}(a - b) + \frac{1}{2}(-a + b + ka)$

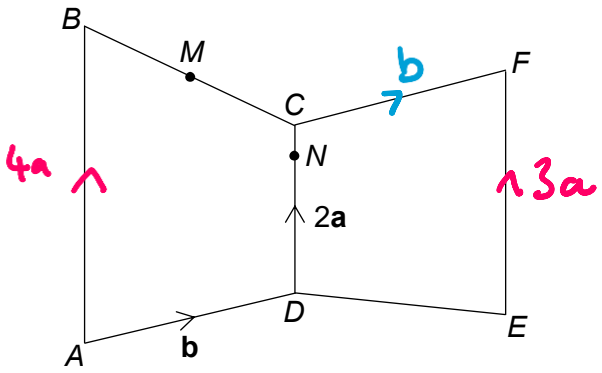
$= \frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b + \frac{k}{2}a$

$= \frac{k}{2}a$

$\overrightarrow{MN} = \frac{k}{2}\overrightarrow{OA}$

therefore they are parallel

12 *ABCD* and *CDEF* are trapeziums



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$\overrightarrow{DC} = 2\mathbf{a}$ $\overrightarrow{AD} = \overrightarrow{CF} = \mathbf{b}$

AB : *DC* : *EF* = 4 : 2 : 3
M is the midpoint of *BC*.
N is on the line *CD*.

MNE is a straight line.

DN : *NC* = k : 1, where k is an integer.

Work out the value of k. **[5 marks]**

$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN}$
$= \frac{1}{2}(\overrightarrow{BC}) + \overrightarrow{CN}$
$= \frac{1}{2}(-4\mathbf{a} + \mathbf{b} + 2\mathbf{a}) - \alpha(2\mathbf{a})$
$= \frac{1}{2}\mathbf{b} - \mathbf{a} - 2\alpha\mathbf{a}$
$= \frac{1}{2}\mathbf{b} - (2\alpha + 1)\mathbf{a}$

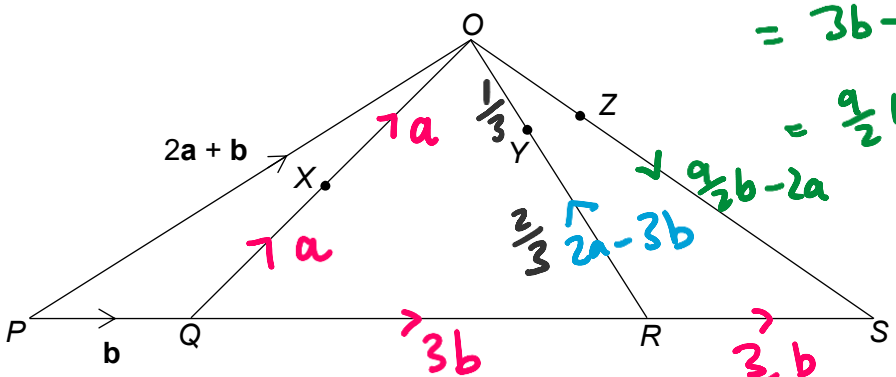
$\overrightarrow{NE} = \overrightarrow{NC} + \overrightarrow{CF} + \overrightarrow{FE}$
$= \alpha(2\mathbf{a}) + \mathbf{b} - 3\mathbf{a}$
$= \mathbf{b} - (3 - 2\alpha)\mathbf{a}$

$\overrightarrow{NE} = 2\overrightarrow{MN}$
$3 - 2\alpha = 2(2\alpha + 1)$
$3 - 2\alpha = 4\alpha + 2$
$\alpha = \frac{1}{6}$

so $NC = \frac{1}{6}DC$
$DC : NC = 6 : 1$
$DN : NC = 5 : 1$

$k =$ 5

13 *POS* is a triangle.



$\overrightarrow{PQ} = \mathbf{b}$ $\overrightarrow{PO} = 2\mathbf{a} + \mathbf{b}$

X is the midpoint of QO
$OY : YR = 1 : 2$
$PQ : QR : RS = 2 : 6 : 3$

XYZ is a straight line.

$OZ : OS = 1 : k$

Work out the value of k. **[6 marks]**

$\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS}$
$= 3b - 2a + \frac{3}{2}b$
$= \frac{9}{2}b - 2a$

$\overrightarrow{RO} = \overrightarrow{RQ} + \overrightarrow{QO}$
$= -3b + 2a$
$= 2a - 3b$

$\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$
$= a - \frac{1}{3}(2a - 3b)$
$= a - \frac{2}{3}a + b$
$= b + \frac{1}{3}a$

$\overrightarrow{XZ} = n(\overrightarrow{XY})$
$= n(b + \frac{1}{3}a)$
$= nb + \frac{n}{3}a$
$1 - \frac{2}{k} = \frac{n}{3}$
$3 - \frac{6}{k} = n$
and $\frac{9}{2k} = n$

$\overrightarrow{XZ} = \overrightarrow{XO} + \overrightarrow{OZ}$
$= a + \frac{1}{k}(\frac{9}{2}b - 2a)$
$= a + \frac{9}{2k}b - \frac{2}{k}a$
$= (1 - \frac{2}{k})a + \frac{9}{2k}b$

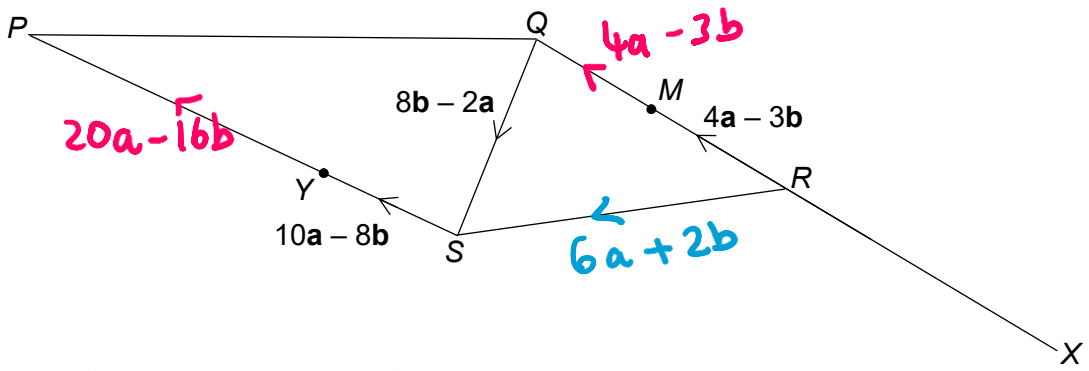
equate

$3 - \frac{6}{k} = \frac{9}{2k}$
$3k - 6 = \frac{9}{2}$
$6k - 12 = 9$
$k = \frac{21}{6} = \frac{7}{2}$

$k =$ 3.5

11

14 *PQRS* is a quadrilateral.



$\overrightarrow{SY} = 10\mathbf{a} - 8\mathbf{b}$ $\overrightarrow{QS} = 8\mathbf{b} - 2\mathbf{a}$ $\overrightarrow{RM} = 4\mathbf{a} - 3\mathbf{b}$

$RM = MQ$
$SY : YP = 1 : 2$
QRX is a straight line.
XS is parallel to RP.

Work out $XS : RP$
Give your answer in the form $n : 1$ **[6 marks]**

$\overrightarrow{RP} = \overrightarrow{RQ} + \overrightarrow{QS} + \overrightarrow{SP}$

$= 8a - 6b + 8b - 2a + 30a - 24b$

$= 36a - 22b$

$\overrightarrow{XS} = \overrightarrow{XR} + \overrightarrow{RS}$ $\overrightarrow{XS} = n\overrightarrow{RP}$

$= K(4a - 3b) + 6a + 2b$ $= 36n\,a - 22n\,b$

$= (4K + 6)a - (3K - 2)b$

$4K + 6 = 36n$ (×3)

$3K - 2 = 22n$ (×4)

$12K + 18 = 108n$

$12K - 8 = 88n$

$26 = 20n$

$n = \frac{26}{20}$

$n = \frac{13}{10}$

Answer 1·3 : 1

6