

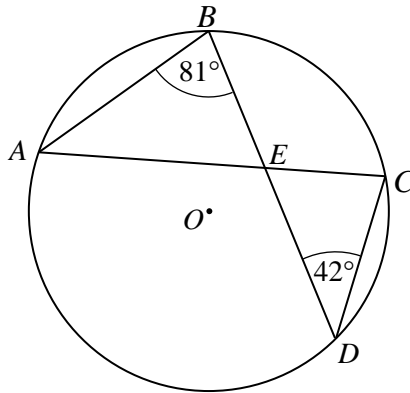


# Circle Theorems



REVISE THIS TOPIC

1 A, B, C and D are points on the circumference of a circle with centre O.



Angle  $ABD = 81^\circ$

Angle  $BDC = 42^\circ$

(a) Work out the size of angle  $CAB$ .

42  
.....  
(1)

(b) Work out the size of angle  $ACD$ .

81  
.....  
(1)

(c) Work out the size of angle  $AEB$ .

57  
.....  
(1)

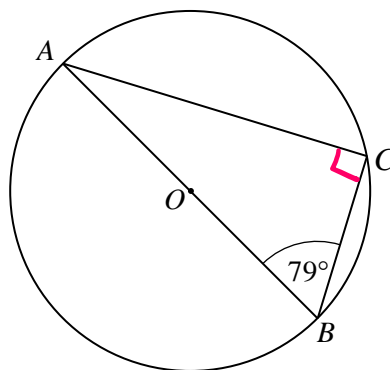
(d) Work out the size of angle  $BEC$ .

123  
.....  
(1)

(Total for Question 1 is 4 marks)



2 A, B and C are points on the circumference of a circle with centre O.



Angle  $ABC = 79^\circ$

(a) Work out the size of angle  $CAB$ .

11  
 .....  
 (1)

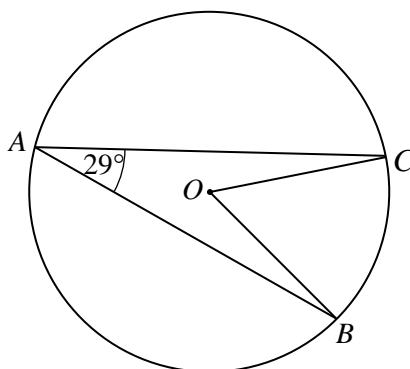
(b) Give a reason for your answer to part (a)

The angle in a semicircle is  $90^\circ$   
 Angles in a triangle add to  $180^\circ$

(1)

(Total for Question 2 is 2 marks)

3 A, B and C are points on the circumference of a circle with centre O.



Angle  $CAB = 29^\circ$

(a) Work out the size of angle  $COB$ .

58  
 .....  
 (1)

(b) Give a reason for your answer to part (a)

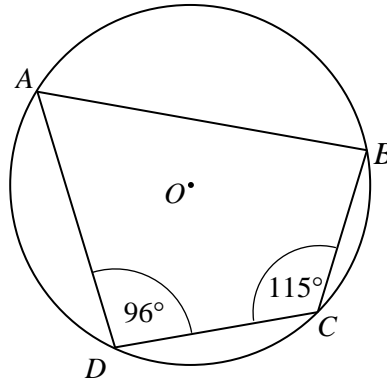
The angle at the centre is twice the angle at the circumference.

(1)

(Total for Question 3 is 2 marks)



4  $A, B, C$  and  $D$  are points on the circumference of a circle with centre  $O$ .



Angle  $ADC = 96^\circ$   
 Angle  $BCD = 115^\circ$

(a) Work out the size of angle  $ABC$ .

84  
 .....  
 (1)

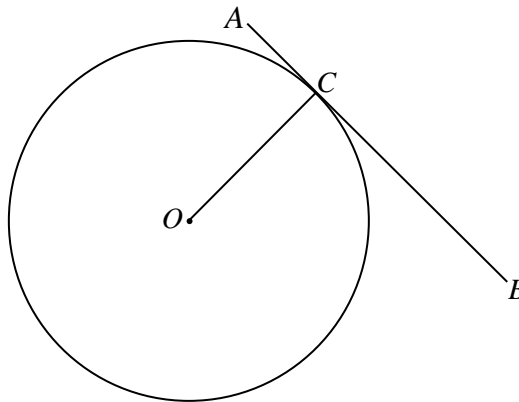
(b) Give a reason for your answer to part (a)

Opposite angles in a cyclic quadrilateral  
 add to  $180^\circ$

.....  
 .....  
 (1)

(Total for Question 4 is 2 marks)

5  $A, B,$  and  $C$  are points on the circumference of a circle with centre  $O$ .  
 $AB$  is the tangent to the circle at point  $C$ .



(a) Write down the size of angle  $OCB$ .

90  
 .....  
 (1)

(b) Give a reason for your answer to part (a)

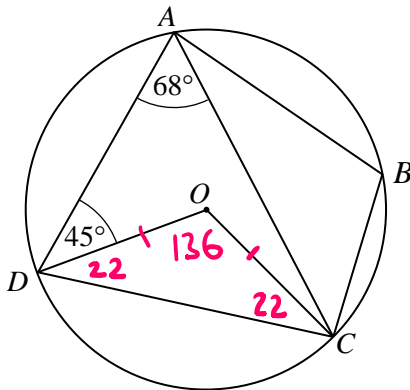
A tangent meets a radius at  $90^\circ$

.....  
 .....  
 (1)

(Total for Question 5 is 2 marks)



6  $A, B, C$  and  $D$  are points on the circumference of a circle with centre  $O$ .



Angle  $DAC = 68^\circ$

Angle  $ADO = 45^\circ$

Work out the size of angle  $ABC$ .

Give reasons for each stage of your working.

Angle  $DOC = 136^\circ$

The angle at the centre is twice the angle at the circumference.

Angle  $ODC = \text{Angle } OCD = 22^\circ$

Base angles in an isosceles triangle are equal

Angle  $ADC = 22 + 45$   
 $= 67$

Angle  $ABC = 180 - 67$   
 $= 113$

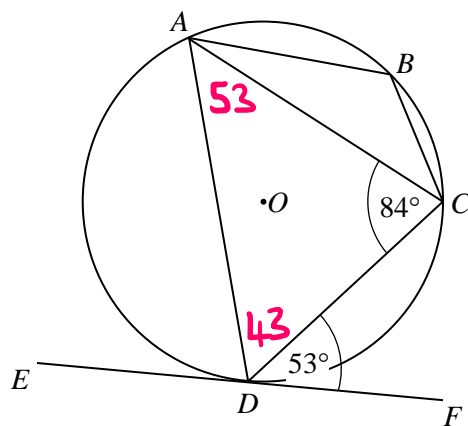
Opposite angles in a cyclic quadrilateral add to  $180^\circ$

113

(Total for Question 6 is 4 marks)



- 7  $A, B, C$  and  $D$  are points on the circumference of a circle with centre  $O$ .  
 $EF$  is the tangent to the circle at point  $D$ .



Angle  $ACD = 84^\circ$   
 Angle  $CDF = 53^\circ$

Work out the size of angle  $ABC$ .  
 Give reasons for each stage of your working.

Angle  $DAC = 53^\circ$

Alternate segment theorem

Angle  $ADC = 43^\circ$

Angles in a triangle add to  $180^\circ$

$$\text{Angle } ABC = 180 - 43$$

$$= 137^\circ$$

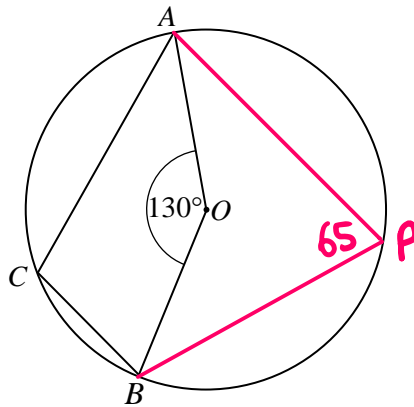
Opposite angles in a cyclic quadrilateral add to  $180^\circ$

137

(Total for Question 7 is 4 marks)



8 A, B, and C are points on the circumference of a circle with centre O.



Angle  $AOB = 130^\circ$

Work out the size of angle  $ACB$ .  
Give reasons for each stage of your working.

Angle  $APB = 65^\circ$

The angle at the centre is twice the angle at the circumference.

Angle  $ABC = 180 - 65$   
 $= 115$

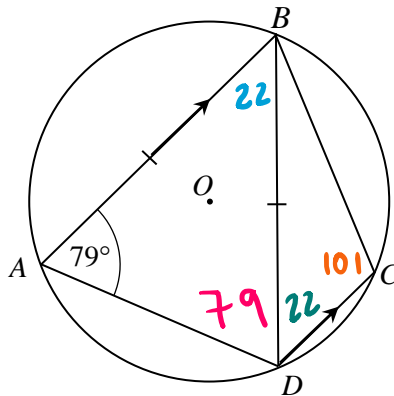
Opposite angles in a cyclic quadrilateral add to  $180^\circ$

115

(Total for Question 8 is 3 marks)



9 A, B, C and D are points on the circumference of a circle with centre O.



Angle  $BAD = 79^\circ$

$BA = BD$

Lines  $AB$  and  $DC$  are parallel.

Work out the size of angle  $DBC$ .

Give reasons for each stage of your working.

Angle  $BAD = \text{Angle } BDA = 79^\circ$

Base angles in an isosceles triangle are equal

Angle  $ABD = 22^\circ$

Angles in a triangle add to  $180^\circ$

Angle  $BDC = \text{Angle } ABD$

Alternate angles are equal

Angle  $ABC = 180 - 43$   
 $= 137^\circ$

Opposite angles in a cyclic quadrilateral add to  $180^\circ$

Angle  $DBC = 57^\circ$

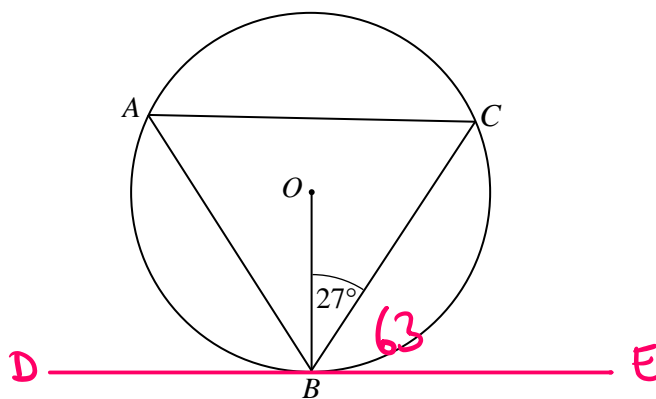
Angles in a triangle add to  $180^\circ$

57

(Total for Question 9 is 5 marks)



10  $A, B$  and  $C$  are points on the circumference of a circle with centre  $O$ .



Angle  $CBO = 27^\circ$

Work out the size of angle  $BAC$ .  
Give reasons for each stage of your working.

$$\begin{aligned} \text{Angle } CBE &= 90 - 27 \\ &= 63^\circ \end{aligned}$$

A tangent meets a radius at  $90^\circ$

$$\begin{aligned} \text{Angle } BAC &= 63^\circ \\ \text{Alternate segment theorem} \end{aligned}$$

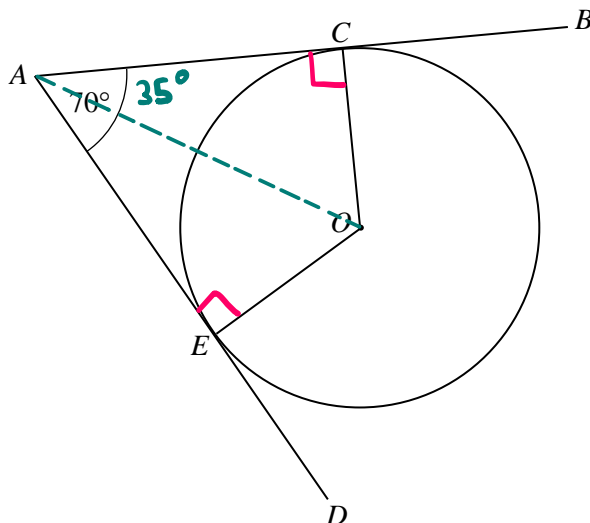
63

(Total for Question 10 is 3 marks)





11  $C$  and  $E$  are points on the circumference of a circle with centre  $O$ .



$AB$  and  $AD$  are tangents to the circle at  $C$  and  $E$ .  
Angle  $EAC = 70^\circ$

(a) Work out the size of angle  $COE$ .

Angle  $AEO = \text{Angle } ACO = 90^\circ$

A tangent meets a radius at  $90^\circ$

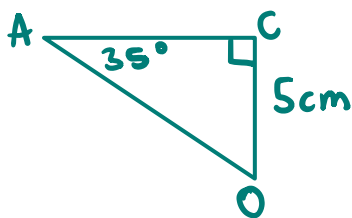
Angle  $COE = 110^\circ$  angles in a quadrilateral add to  $360^\circ$

$OC = 5 \text{ cm}$

.....  
110  
(2)

(b) Work out the length of  $CA$ .  
Give your answer to 1 decimal place.

Angle  $CAO = 35^\circ$



$\tan(35) = \frac{5}{CA}$

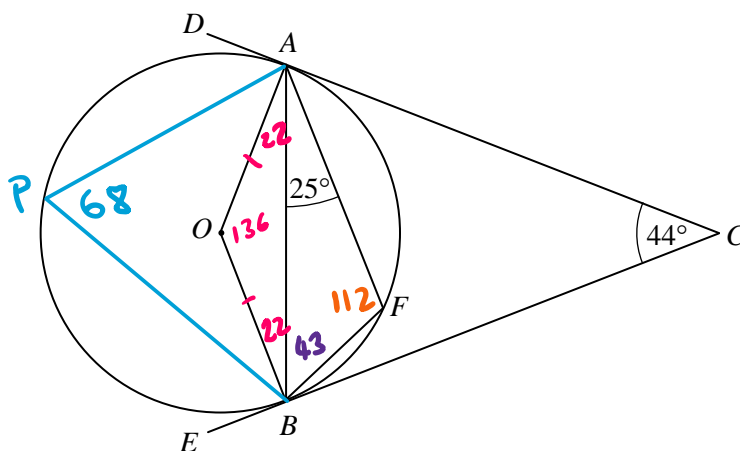
$CA = \frac{5}{\tan(35)}$

$CA = 7.407\dots$  ..... 7.1 ..... cm  
(2)

(Total for Question 11 is 4 marks)



12 A and B are points on the circumference of a circle with centre O.



DC and EC are tangents to the circle at A and B.

Angle  $ACB = 44^\circ$

Angle  $BAF = 25^\circ$

[Reasons not required in this Q]

Work out the size of angle  $FBC$ .

Angle  $AOB = 136^\circ$  A tangent meets a radius at  $90^\circ$   
and angles in a quadrilateral add to  $360^\circ$

Angle  $ABO = \text{Angle } BAO = 22^\circ$

Base angles in an isosceles triangle are equal

Angle  $APB = 68^\circ$

The angle at the centre is twice the angle at the circumference.

Angle  $AFB = 112^\circ$

Opposite angles in a cyclic quadrilateral add to  $180^\circ$

Angle  $ABF = 43^\circ$  Angles in a triangle add to  $180^\circ$

Angle  $FBC = 25^\circ$

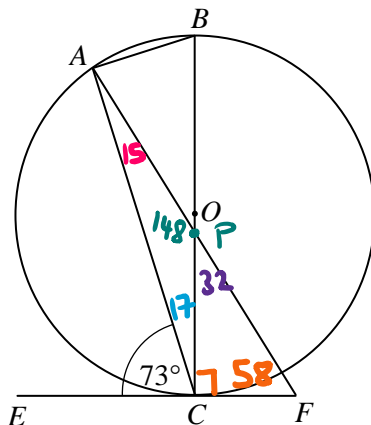
A tangent meets a radius at  $90^\circ$

25

(Total for Question 12 is 4 marks)



13  $A, B$  and  $C$  are points on the circumference of a circle with centre  $O$ .



$EF$  is the tangent to the circle at  $C$ .

Angle  $ACE = 73^\circ$

Angle  $FAB = 5 \times$  Angle  $CAF$

Work out the size of angle  $AFC$ .

[Reasons not required in this Q]

Angle  $ACB = 90^\circ$  The angle in a semicircle is  $90^\circ$

$90 \div 6 = 15$

Angle  $CAF = 15^\circ$

Angle  $ECB = 90^\circ$  Angle  $ACB = 17^\circ$

A tangent meets a radius at  $90^\circ$

Angle  $APC = 148^\circ$  Angles in a triangle add to  $180^\circ$

Angle  $CPF = 32^\circ$  Angles on a straight line add to  $180^\circ$

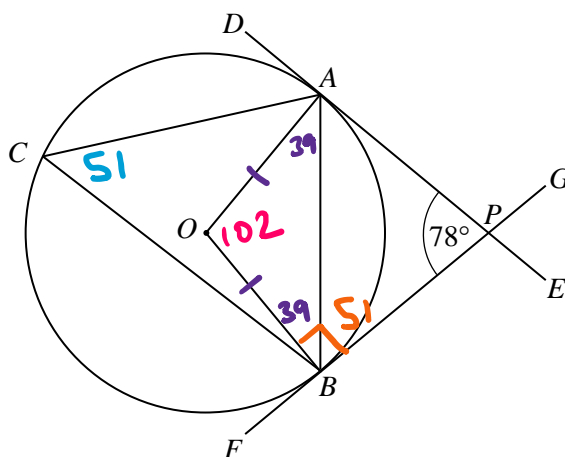
Angle  $AFC = 58^\circ$  Angles in a triangle add to  $180^\circ$

58

(Total for Question 13 is 4 marks)



14 A, B and C are points on the circumference of a circle with centre O.



DE and FG are tangents to the circle at A and B that intersect at the point P.  
Angle APB =  $78^\circ$

[Reasons not required in this Q]

(a) Work out the size of angle ACB.

Angle AOB =  $102^\circ$  A tangent meets a radius at  $90^\circ$   
and angles in a quadrilateral add to  $360^\circ$

Angle ACB =  $51^\circ$  Angle at the circumference is  
half the angle at the centre

51

(b) Work out the size of angle ABP.

Angle ABO = Angle BAO =  $39^\circ$

Base angles in an isosceles triangle are equal

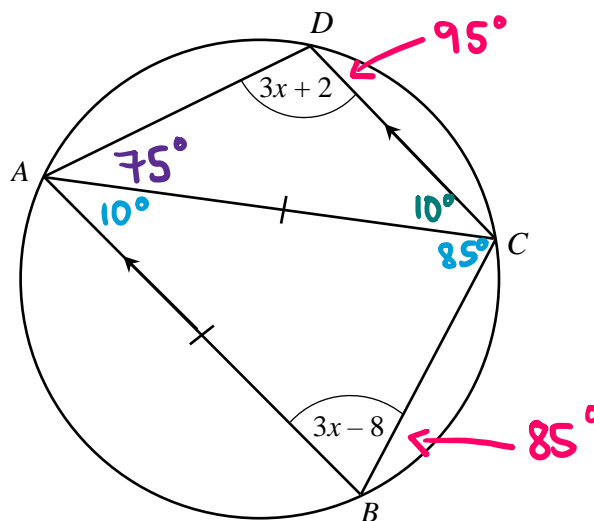
Angle ABP =  $90 - 39$   
= 51

A tangent meets a radius at  $90^\circ$

51



15



$A, B, C$  and  $D$  are points on the circumference of a circle.  
 $ABCD$  is a trapezium with  $AB$  parallel to  $DC$ .

$AB = AC$

[Reasons not required in this Q]

Work out the size of angle  $DAC$ .  
 You must show all your working.

$3x + 2 + 3x - 8 = 180^\circ$

Opposite angles in a cyclic quadrilateral add to  $180^\circ$

$6x - 6 = 180$

$3(31) + 2 = 95$

$6x = 186$

$3(31) - 8 = 85$

$x = 31^\circ$

Angle  $ACB =$  Angle  $ABC = 85$

Angle  $CAB = 10^\circ$  Angles in a triangle add to  $180^\circ$

Angle  $DCA =$  Angle  $CAB$  alternate angles are equal

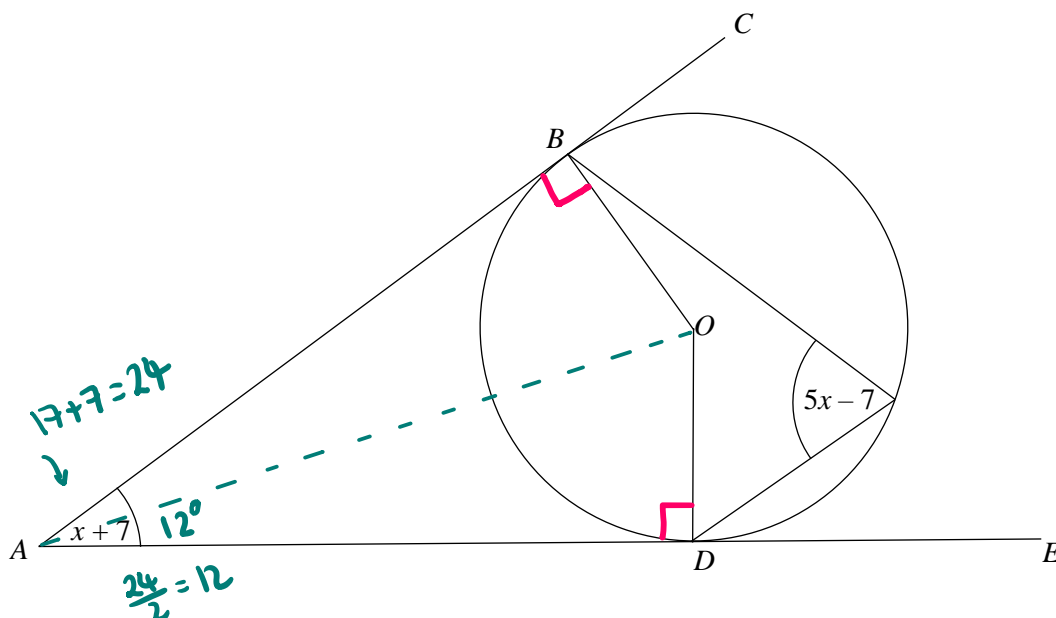
Angle  $DAC = 75^\circ$  Angles in a triangle add to  $180^\circ$

75

(Total for Question 15 is 5 marks)



16



$B$  and  $D$  are points on the circumference of a circle, centre  $O$ .  
 $ABC$  and  $ADE$  are tangents to the circle.

[Reasons not required in this Q]

- (a) Work out value of  $x$ .  
 You must show all your working.

$\text{Angle } BOD = 2(5x - 7)$   
 (twice angle at circumference)

$\text{Angle } BOD = 180 - (x + 7)$   
 (angles in quadrilateral add to  $360^\circ$ )

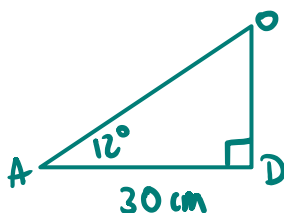
$10x - 14 = 180 - x - 7$   
 $10x - 14 = 173 - x$   
 $11x = 187$   
 $x = 17$

$x = \underline{\quad 17 \quad}$   
 (3)

$AD = 30$  cm

- (b) Work out the radius of the circle.  
 Give your answer to three significant figures.

$\angle BAD = 24^\circ$   
 $\angle OAD = 12^\circ$



$\tan(12) = \frac{OD}{30}$   
 $OD = 30 \tan(12)$

$\underline{\quad 3.38 \quad}$  cm  
 (3)

(Total for Question 17 is 6 marks)

